Additional Topics in Math

In addition to the questions in Heart of Algebra, Problem Solving and Data Analysis, and Passport to Advanced Math, the SAT Math Test includes several questions that are drawn from areas of geometry, trigonometry, and the arithmetic of complex numbers. They include both multiple-choice and student-produced response questions. Some of these questions appear in the no-calculator portion, where the use of a calculator is not permitted, and others are in the calculator portion, where the use of a calculator is permitted.

Let's explore the content and skills assessed by these questions.

Geometry

The SAT Math Test includes questions that assess your understanding of the key concepts in the geometry of lines, angles, triangles, circles, and other geometric objects. Other questions may also ask you to find the area, surface area, or volume of an abstract figure or a real-life object. You don't need to memorize a large collection of formulas, but you should be comfortable understanding and using these formulas to solve various types of problems. Many of the geometry formulas are provided in the reference information at the beginning of each section of the SAT Math Test, and less commonly used formulas required to answer a question are given with the question.

To answer geometry questions on the SAT Math Test, you should recall the geometry definitions learned prior to high school and know the essential concepts extended while learning geometry in high school. You should also be familiar with basic geometric notation.

Here are some of the areas that may be the focus of some questions on the SAT Math Test.

- Lines and angles
  - Lengths and midpoints
  - Measures of angles
  - Vertical angles
  - Angle addition
  - Straight angles and the sum of the angles about a point

REMEMBER

Six of the 58 questions (approximately 10%) on the SAT Math Test will be drawn from Additional Topics in Math, which includes geometry, trigonometry, and the arithmetic of complex numbers.

REMEMBER

You do not need to memorize a large collection of geometry formulas. Many geometry formulas are provided on the SAT Math Test in the Reference section of the directions.

- ⬥ Properties of parallel lines and the angles formed when parallel lines are cut by a transversal

- ⬥ Properties of perpendicular lines

- ▪ Triangles and other polygons

  - ⬥ Right triangles and the Pythagorean theorem

  - ⬥ Properties of equilateral and isosceles triangles

  - ⬥ Properties of 30°-60°-90° triangles and 45°-45°-90° triangles

  - ⬥ Congruent triangles and other congruent figures

  - ⬥ Similar triangles and other similar figures

  - ⬥ The triangle inequality

  - ⬥ Squares, rectangles, parallelograms, trapezoids, and other quadrilaterals

  - ⬥ Regular polygons

- ▪ Circles

  - ⬥ Radius, diameter, and circumference

  - ⬥ Measure of central angles and inscribed angles

  - ⬥ Arc length, arc measure, and area of sectors

  - ⬥ Tangents and chords

- ▪ Area and volume

  - ⬥ Area of plane figures

  - ⬥ Volume of solids

  - ⬥ Surface area of solids

You should be familiar with the geometric notation for points and lines, line segments, angles and their measures, and lengths.
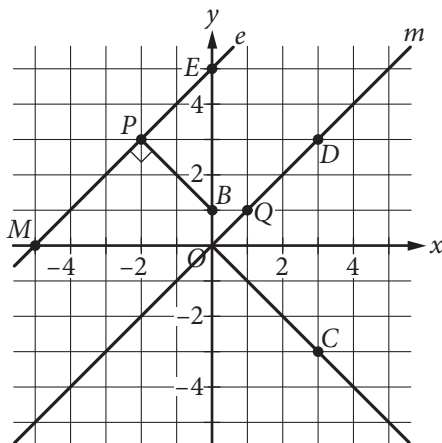
In the figure above, the *xy*-plane has origin *O*. The values of *x* on the horizontal *x*-axis increase as you move to the right, and the values of *y* on the vertical *y*-axis increase as you move up. Line *e* contains point *P*,

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The triangle inequality theorem states that for any triangle, the length of any side of the triangle must be less than the sum of the lengths of the other two sides of the triangle and greater than the difference of the lengths of the other two sides.

which has coordinates (−2, 3); point *E*, which has coordinates (0, 5); and point *M*, which has coordinates (−5, 0). Line *m* passes through the origin *O* (0, 0), the point *Q* (1, 1), and the point *D* (3, 3).

Lines *e* and *m* are parallel—they never meet. This is written *e* ∥ *m*.

You will also need to know the following notation:

- $\overleftrightarrow{PE}$: the line containing the points *P* and *E* (this is the same as line *e*)
- $\overline{PE}$ or line segment *PE*: the line segment with endpoints *P* and *E*
- *PE*: the length of segment *PE* (you can write *PE* = 2√2)
- $\overrightarrow{PE}$: the ray starting at point *P* and extending indefinitely in the direction of point *E*
- $\overrightarrow{EP}$: the ray starting at point *E* and extending indefinitely in the direction of point *P*
- ∠*DOC*: the angle formed by $\overrightarrow{OD}$ and $\overrightarrow{OC}$
- △*PEB*: the triangle with vertices *P*, *E*, and *B*
- Quadrilateral *BPMO*: the quadrilateral with vertices *B*, *P*, *M*, and *O*
- $\overline{BP} \perp \overline{PM}$: segment *BP* is perpendicular to segment *PM* (you should also recognize that the right angle box within ∠*BPM* means this angle is a right angle)

Example 1
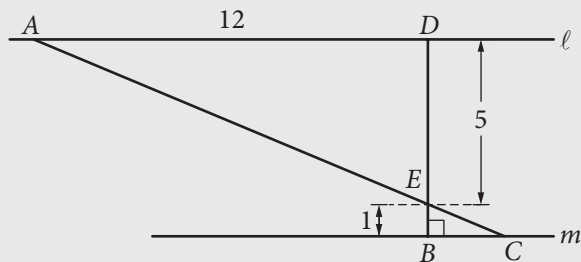
In the figure above, line ℓ is parallel to line *m*, segment *BD* is perpendicular to line *m*, and segment *AC* and segment *BD* intersect at *E*. What is the length of segment *AC*?

Since segment *AC* and segment *BD* intersect at *E*, ∠*AED* and ∠*CEB* are vertical angles, and so the measure of ∠*AED* is equal to the measure of ∠*CEB*. Since line ℓ is parallel to line *m*, ∠*BCE* and ∠*DAE* are alternate interior angles of parallel lines cut by a transversal, and so the measure of ∠*BCE* is equal to the measure of ∠*DAE*. By the angle-angle theorem, △*AED* is similar to △*CEB*, with vertices *A*, *E*, and *D* corresponding to vertices *C*, *E*, and *B*, respectively.

Also, △*AED* is a right triangle, so by the Pythagorean theorem, $AE = \sqrt{AD^2 + DE^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$. Since △*AED* is similar to △*CEB*, the ratios of the lengths of corresponding sides of the two

triangles are in the same proportion, which is $\frac{ED}{EB} = \frac{5}{1} = 5$. Thus, $\frac{AE}{EC} = \frac{13}{EC} = 5$, and so $EC = \frac{13}{5}$. Therefore, $AC = AE + EC = 13 + \frac{13}{5} = \frac{78}{5}$.

Note some of the key concepts that were used in Example 1:

- Vertical angles have the same measure.

- When parallel lines are cut by a transversal, the alternate interior angles have the same measure.

- If two angles of a triangle are congruent to (have the same measure as) two angles of another triangle, the two triangles are similar.

- The Pythagorean theorem: $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.

- If two triangles are similar, then all ratios of lengths of corresponding sides are equal.

- If point E lies on line segment AC, then $AC = AE + EC$.

Note that if two triangles or other polygons are similar or congruent, the order in which the vertices are named does *not* necessarily indicate how the vertices correspond in the similarity or congruence. Thus, it was stated explicitly in Example 1 that "$\triangle AED$ is similar to $\triangle CEB$, with vertices A, E, and D corresponding to vertices C, E, and B, respectively."

You should also be familiar with the symbols for congruence and similarity.

- Triangle ABC is congruent to triangle DEF, with vertices A, B, and C corresponding to vertices D, E, and F, respectively, and can be written as $\triangle ABC \cong \triangle DEF$. Note that this statement, written with the symbol $\cong$, indicates that vertices A, B, and C correspond to vertices D, E, and F, respectively.

- Triangle ABC is similar to triangle DEF, with vertices A, B, and C corresponding to vertices D, E, and F, respectively, and can be written as $\triangle ABC \sim \triangle DEF$. Note that this statement, written with the symbol $\sim$, indicates that vertices A, B, and C correspond to vertices D, E, and F, respectively.

Example 2
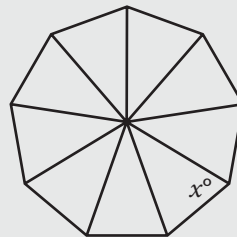
In the figure above, a regular polygon with 9 sides has been divided into 9 congruent isosceles triangles by line segments drawn from the center of the polygon to its vertices. What is the value of x?

The sum of the measures of the angles around a point is 360°. Since the 9 triangles are congruent, the measures of each of the 9 angles are equal. Thus, the measure of each of the 9 angles around the center point is $\frac{360°}{9}$ = 40°. In any triangle, the sum of the measures of the interior angles is 180°. So in each triangle, the sum of the measures of the remaining two angles is 180° − 40° = 140°. Since each triangle is isosceles, the measure of each of these two angles is the same. Therefore, the measure of each of these angles is $\frac{140°}{2}$ = 70°. Hence, the value of x is 70.

Note some of the key concepts that were used in Example 2:

- The sum of the measures of the angles about a point is 360°.

- Corresponding angles of congruent triangles have the same measure.

- The sum of the measure of the interior angles of any triangle is 180°.

- In an isosceles triangle, the angles opposite the sides of equal length are of equal measure.

Example 3
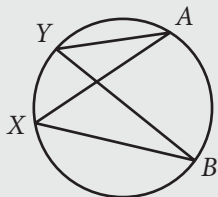
In the figure above, $\angle AXB$ and $\angle AYB$ are inscribed in the circle. Which of the following statements is true?

A) The measure of $\angle AXB$ is greater than the measure of $\angle AYB$.

B) The measure of $\angle AXB$ is less than the measure of $\angle AYB$.

C) The measure of $\angle AXB$ is equal to the measure of $\angle AYB$.

D) There is not enough information to determine the relationship between the measure of $\angle AXB$ and the measure of $\angle AYB$.

Choice C is correct. Let the measure of arc $\overparen{AB}$ be $d°$. Since $\angle AXB$ is inscribed in the circle and intercepts arc $\overparen{AB}$, the measure of $\angle AXB$ is equal to half the measure of arc $\overparen{AB}$. Thus, the measure of $\angle AXB$ is $\frac{d°}{2}$. Similarly, since $\angle AYB$ is also inscribed in the circle and intercepts arc $\overparen{AB}$, the measure of $\angle AYB$ is also $\frac{d°}{2}$. Therefore, the measure of $\angle AXB$ is equal to the measure of $\angle AYB$.

Note the key concept that was used in Example 3:

- The measure of an angle inscribed in a circle is equal to half the measure of its intercepted arc.

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At first glance, it may appear as though there's not enough information to determine the relationship between the two angle measures. One key to this question is identifying what is the same about the two angle measures. In this case, both angles intercept arc $\overparen{AB}$.

You also should know these related concepts:

- The measure of a central angle in a circle is equal to the measure of its intercepted arc.

- An arc is measured in degrees, while arc length is measured in linear units.

You should also be familiar with notation for arcs and circles on the SAT:

- A circle may be identified by the point at its center; for instance, "the circle centered at point *M*" or "the circle with center at point *M*."

- An arc named with only its two endpoints, such as arc $\overset{\frown}{AB}$, will always refer to a minor arc. A minor arc has a measure that is less than 180°.

- An arc may also be named with three points: the two endpoints and a third point that the arc passes through. So, arc $\overset{\frown}{ACB}$ has endpoints at *A* and *B* and passes through point *C*. Three points may be used to name a minor arc or an arc that has a measure of 180° or more.

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REMEMBER

Figures are drawn to scale on the SAT Math Test unless explicitly stated otherwise. If a question states that a figure is not drawn to scale, be careful not to make unwarranted assumptions about the figure.

In general, figures that accompany questions on the SAT Math Test are intended to provide information that is useful in answering the question. They are drawn as accurately as possible EXCEPT in a particular question when it is stated that the figure is not drawn to scale. In general, even in figures not drawn to scale, the relative positions of points and angles may be assumed to be in the order shown. Also, line segments that extend through points and appear to lie on the same line may be assumed to be on the same line. A point that appears to lie on a line or curve may be assumed to lie on the line or curve.

The text "Note: Figure not drawn to scale." is included with the figure when degree measures may not be accurately shown and specific lengths may not be drawn proportionally. The following example illustrates what information can and cannot be assumed from a figure not drawn to scale.
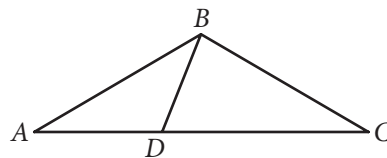
Note: Figure not drawn to scale.

A question may refer to a triangle such as *ABC* above. Although the note indicates that the figure is not drawn to scale, you may assume the following from the figure:

- *ABD* and *DBC* are triangles.

- *D* is between *A* and *C*.

- *A*, *D*, and *C* are points on a line.

- The length of $\overline{AD}$ is less than the length of $\overline{AC}$.

- The measure of angle *ABD* is less than the measure of angle *ABC*.

You may *not* assume the following from the figure:

- The length of $\overline{AD}$ is less than the length of $\overline{DC}$.

- The measures of angles *BAD* and *DBA* are equal.

- The measure of angle *DBC* is greater than the measure of angle *ABD*.

- Angle *DBC* is a right angle.

Example 4
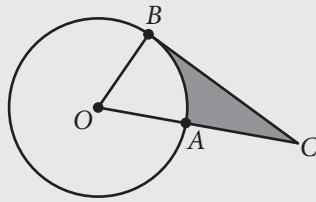
In the given figure, *O* is the center of the circle, segment *BC* is tangent to the circle at *B*, and *A* lies on segment *OC*. If *OB* = *AC* = 6, what is the area of the shaded region?

A) $18\sqrt{3} - 3\pi$

B) $18\sqrt{3} - 6\pi$

C) $36\sqrt{3} - 3\pi$

D) $36\sqrt{3} - 6\pi$

Since segment *BC* is tangent to the circle at *B*, it follows that $\overline{BC} \perp \overline{OB}$, and so triangle *OBC* is a right triangle with its right angle at *B*. Since *OB* = 6 and *OB* and *OA* are both radii of the circle, *OA* = *OB* = 6, and *OC* = *OA* + *AC* = 12. Thus, triangle *OBC* is a right triangle with the length of the hypotenuse (*OC* = 12) twice the length of one of its legs (*OB* = 6). It follows that triangle *OBC* is a 30°-60°-90° triangle with its 30° angle at *C* and its 60° angle at *O*. The area of the shaded region is the area of triangle *OBC* minus the area of the sector bounded by radii *OA* and *OB*.

In the 30°-60°-90° triangle *OBC*, the length of side *OB*, which is opposite the 30° angle, is 6. Thus, the length of side *BC*, which is opposite the 60° angle, is $6\sqrt{3}$. Hence, the area of triangle *OBC* is $\frac{1}{2}(6)(6\sqrt{3}) = 18\sqrt{3}$. Since the sector bounded by radii *OA* and *OB* has central angle 60°, the area of this sector is $\frac{60}{360} = \frac{1}{6}$ of the area of the circle. Since the circle has radius 6, its area is $\pi(6)^2 = 36\pi$, and so the area of the sector is $\frac{1}{6}(36\pi) = 6\pi$. Therefore, the area of the shaded region is $18\sqrt{3} - 6\pi$, which is choice B.

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On complex multistep questions such as Example 4, start by identifying the task (finding the area of the shaded region) and considering the intermediate steps that you'll need to solve for (the area of triangle *OBC* and the area of sector *OBA*) in order to get to the final answer. Breaking up this question into a series of smaller questions will make it more manageable.

Note some of the key concepts that were used in Example 4:

- A tangent to a circle is perpendicular to the radius of the circle drawn to the point of tangency.

- Properties of 30°-60°-90° triangles.

- Area of a circle.

- The area of a sector with central angle $x°$ is equal to $\frac{x}{360}$ of the area of the entire circle.

Example 5
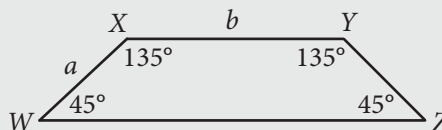
Trapezoid *WXYZ* is shown above. How much greater is the area of this trapezoid than the area of a parallelogram with side lengths *a* and *b* and base angles of measure 45° and 135°?
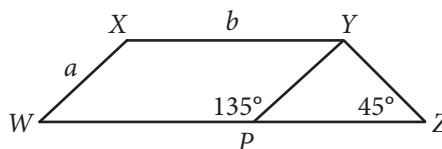
A) $\frac{1}{2}a^2$

B) $\sqrt{2}a^2$

C) $\frac{1}{2}ab$

D) $\sqrt{2}ab$

In the figure, draw a line segment from *Y* to the point *P* on side *WZ* of the trapezoid such that ∠*YPW* has measure 135°, as shown in the figure below.


Since in trapezoid *WXYZ* side *XY* is parallel to side *WZ*, it follows that *WXYP* is a parallelogram with side lengths *a* and *b* and base angles of measure 45° and 135°. Thus, the area of the trapezoid is greater than a parallelogram with side lengths *a* and *b* and base angles of measure 45° and 135° by the area of triangle *PYZ*. Since ∠*YPW* has measure 135°, it follows that ∠*YPZ* has measure 45°. Hence, triangle *PYZ* is a 45°-45°-90° triangle with legs of length *a*. Therefore, its area is $\frac{1}{2}a^2$, which is choice A.

Note some of the key concepts that were used in Example 5:

- Properties of trapezoids and parallelograms

- Area of a 45°-45°-90° triangle

Some questions on the SAT Math Test may ask you to find the area, surface area, or volume of an object, possibly in a real-life context.

Example 6
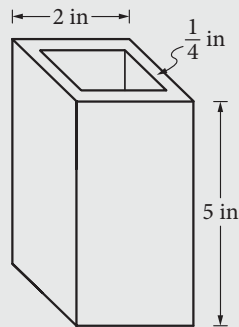
Note: Figure not drawn to scale.

A glass sculpture in the shape of a right square prism is shown. The base of the sculpture's outer shape is a square of side length 2 inches. The sculpture has a hollow core that is also in the shape of a right square prism. The glass in the sculpture is $\frac{1}{4}$ inch thick, and the height of both the glass and the hollow core is 5 inches. What is the volume, in cubic inches, of the glass in the sculpture?

A) 1.50

B) 8.75

C) 11.25

D) 20.00

The volume of the glass in the sculpture can be calculated by subtracting the volume of the inside hollow core from the volume of the outside prism. The inside and outside volumes are square-based prisms of different sizes. The outside dimensions of the prism are 5 inches by 2 inches by 2 inches, so its volume is (5)(2)(2) = 20 cubic inches. Each side of the sculpture is $\frac{1}{4}$ inch thick, so each side length of the inside volume is $2 - \frac{1}{4} - \frac{1}{4} = \frac{3}{2}$, or 1.5 inches. Thus, the inside volume of the hollow core is (5)(1.5)(1.5) = 11.25 cubic inches. Therefore, the volume of the glass in the sculpture is 20 – 11.25 = 8.75 cubic inches, which is choice B.

Coordinate Geometry

Questions on the SAT Math Test may ask you to use the coordinate plane and equations of lines and circles to describe figures. You may be asked to create the equation of a circle given the figure or use the structure of a given equation to determine a property of a

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Pay close attention to detail on a question such as Example 6. You must take into account the fact that the sculpture has a hollow core that is also a right square prism, with the same height but a different-sized base, when subtracting the volume of the hollow core from the volume of the entire sculpture.

figure in the coordinate plane. You should know that the graph of $(x - a)^2 + (y - b)^2 = r^2$ in the xy-plane is a circle with center (a, b) and radius r.

Example 7

$$x^2 + (y + 1)^2 = 4$$

The graph of the given equation in the xy-plane is a circle. If the center of this circle is translated 1 unit up and the radius is increased by 1, which of the following is an equation of the resulting circle?

A) $x^2 + y^2 = 5$

B) $x^2 + y^2 = 9$

C) $x^2 + (y + 2)^2 = 5$

D) $x^2 + (y + 2)^2 = 9$

The graph of the equation $x^2 + (y + 1)^2 = 4$ in the xy-plane is a circle with center $(0, -1)$ and radius $\sqrt{4} = 2$. If the center is translated 1 unit up, the center of the new circle will be $(0, 0)$. If the radius is increased by 1, the radius of the new circle will be 3. Therefore, an equation of the new circle in the xy-plane is $x^2 + y^2 = 3^2 = 9$, so choice B is correct.

Example 8

$$x^2 + 8x + y^2 - 6y = 24$$

The graph of the equation above in the xy-plane is a circle. What is the radius of the circle?

The given equation is not in the standard form $(x - a)^2 + (y - b)^2 = r^2$. You can put it in standard form by completing the square. Since the coefficient of x is 8 and the coefficient of y is -6, you can write the equation in terms of $(x + 4)^2$ and $(y - 3)^2$ as follows:

$$x^2 + 8x + y^2 - 6y = 24$$
$$(x^2 + 8x + 16) - 16 + (y^2 - 6y + 9) - 9 = 24$$
$$(x + 4)^2 - 16 + (y - 3)^2 - 9 = 24$$
$$(x + 4)^2 + (y - 3)^2 = 24 + 16 + 9$$
$$(x + 4)^2 + (y - 3)^2 = 49$$

Since $49 = 7^2$, the radius of the circle is 7. (Also, the center of the circle is $(-4, 3)$.)

Trigonometry and Radians

Questions on the SAT Math Test may ask you to apply the definitions of right triangle trigonometry. You should also know the definition of radian measure; you may also need to convert between angle measure in degrees and radians. You may need to evaluate trigonometric functions at benchmark angle measures such as $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3},$

and $\frac{\pi}{2}$ radians (which are equal to the angle measures 0°, 30°, 45°, 60°, and 90°, respectively). You will *not* be asked for values of trigonometric functions that require a calculator.

For an acute angle, the trigonometric functions sine, cosine, and tangent can be defined using right triangles. (Note that the functions are often abbreviated as sin, cos, and tan, respectively.)
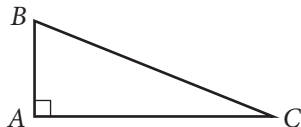
For ∠C in the right triangle above:

- $\sin(\angle C) = \frac{AB}{BC} = \frac{\text{length of leg opposite } \angle C}{\text{length of hypotenuse}}$

- $\cos(\angle C) = \frac{AC}{BC} = \frac{\text{length of leg adjacent to } \angle C}{\text{length of hypotenuse}}$

- $\tan(\angle C) = \frac{AB}{AC} = \frac{\text{length of leg opposite } \angle C}{\text{length of leg adjacent to } \angle C} = \frac{\sin(\angle C)}{\cos(\angle C)}$

The functions will often be written as sin *C*, cos *C*, and tan *C*, respectively.

Note that the trigonometric functions are actually functions of the *measures* of an angle, not the angle itself. Thus, if the measure of ∠C is, say, 30°, you can write sin(30°), cos(30°), and tan(30°), respectively.

Also note that sine and cosine are cofunctions and that

$\sin B = \frac{\text{length of leg opposite } \angle B}{\text{length of hypotenuse}} = \frac{AC}{BC} = \cos C$. This is the

complementary angle relationship: $\sin(x°) = \cos(90° - x°)$.

Example 9
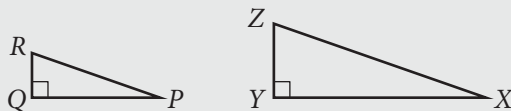
In the figure above, right triangle *PQR* is similar to right triangle *XYZ*, with vertices *P*, *Q*, and *R* corresponding to vertices *X*, *Y*, and *Z*, respectively. If cos *R* = 0.263, what is the value of cos *Z*?

By the definition of cosine, $\cos R = \frac{RQ}{RP}$ and $\cos Z = \frac{ZY}{ZX}$. Since triangle *PQR* is similar to triangle *XYZ*, with vertices *P*, *Q*, and *R* corresponding to vertices *X*, *Y*, and *Z*, respectively, the ratios $\frac{RQ}{RP}$ and $\frac{ZY}{ZX}$ are equal.

Therefore, since $\cos R = \frac{RQ}{RP} = 0.263$, it follows that $\cos Z = \frac{ZY}{ZX} = 0.263$.

Note that this is why, to find the values of the trigonometric functions of, say, *d*°, you can use *any* right triangle with an acute angle of measure *d*° and then take the appropriate ratio of lengths of sides.

Note that since an acute angle of a right triangle has measure between 0° and 90°, exclusive, right triangles can be used only to find values of trigonometric functions for angles with measures between 0° and 90°, exclusive. The definitions of sine, cosine, and tangent can be extended to all values. This is done using radian measure and the unit circle.
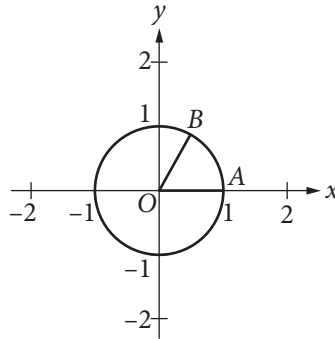
The circle above has radius 1 and is centered at the origin, *O*. An angle in the coordinate plane is said to be in standard position if it meets these two conditions: (1) its vertex lies at the origin and (2) one of its sides lies along the positive *x*-axis. Since ∠*AOB* above, formed by segments *OA* and *OB*, meets both these conditions, it is said to be in *standard position*. As segment *OB*, also called the *terminal side* of ∠*AOB*, rotates counterclockwise about the circle, while *OA* is anchored along the *x*-axis, the *radian* measure of ∠*AOB* is defined to be the length *s* of the arc that ∠*AOB* intercepts on the unit circle. In other words, the measure of ∠*AOB* is *s* radians.

When an acute ∠*AOB* is in standard position within the unit circle, the *x*-coordinate of point *B* is cos(∠*AOB*), and the *y*-coordinate of point *B* is sin(∠*AOB*). When ∠*AOB* is greater than 90 degrees (or $\frac{\pi}{2}$ radians), and point *B* extends beyond the boundaries of the positive *x*-axis and positive *y*-axis, the values of cos(∠*AOB*) and sin(∠*AOB*) may be negative depending on the coordinates of point *B*. For any ∠*AOB*, place ∠*AOB* in standard position within the circle of radius 1 centered at the origin, with side *OA* along the positive *x*-axis and terminal side *OB* intersecting the circle at point *B*. Then the cosine of ∠*AOB* is the *x*-coordinate of *B*, and the sine of ∠*AOB* is the *y*-coordinate of *B*. The tangent of ∠*AOB* is the sine of ∠*AOB* divided by the cosine of ∠*AOB*.

An angle with a full rotation about point *O* has measure 360°. This angle intercepts the full circumference of the circle, which has length 2*π*. Thus, $\frac{\text{measure of an angle in radians}}{\text{measure of an angle in degrees}} = \frac{2\pi}{360°}$. It follows that measure of an angle in radians = $\frac{2\pi}{360°}$ × measure of an angle in degrees and measure of an angle in degrees = $\frac{360°}{2\pi}$ × measure of an angle in radians.

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To convert from degrees to radians, multiply the number of degrees by 2*π* / 360 degrees. To convert from radians to degrees, multiply the number of radians by 360 degrees / 2*π*.

Also note that since a rotation of 2π about point O brings you back to the same point on the unit circle, $\sin(s + 2\pi) = \sin(s)$, $\cos(s + 2\pi) = \cos(s)$, and $\tan(s + 2\pi) = \tan(s)$, for any radian measure s.

Let angle DEF be a central angle in a circle of radius r, as shown in the following figure.
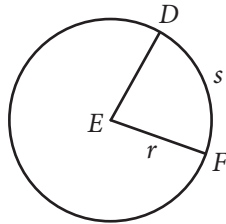
A circle of radius r is similar to a circle of radius 1, with constant of proportionality equal to r. Thus, the length s of the arc intercepted by angle DEF is r times the length of the arc that would be intercepted by an angle of the same measure in a circle of radius 1. Therefore, in the figure above, $s = r \times$ (radian measure of angle DEF), or radian measure of angle $DEF = \frac{s}{r}$.
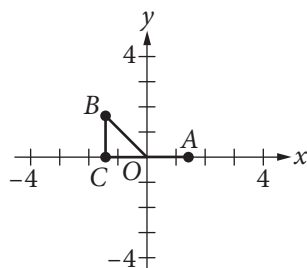
Example 10
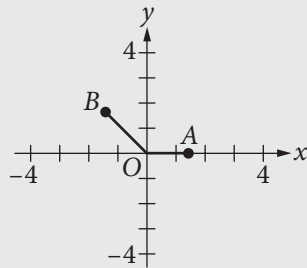
In the figure above, the coordinates of point B are $(-\sqrt{2}, \sqrt{2})$. What is the measure, in radians, of angle AOB?

A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) $\frac{3\pi}{4}$

D) $\frac{5\pi}{4}$

Let C be the point $(-\sqrt{2}, 0)$. Then triangle BOC, shown in the figure below, is a right triangle with both legs of length $\sqrt{2}$.

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Always be on the lookout for special right triangles. Here, noticing that segment OB is the hypotenuse of a 45°-45°-90° triangle makes this question easier to solve.

Hence, triangle *BOC* is a 45°-45°-90° triangle. Thus, angle *COB* has measure 45°, and angle *AOB* has measure 180° − 45° = 135°. Therefore, the measure of angle *AOB* in radians is 135° × $\frac{2\pi}{360°}$ = $\frac{3\pi}{4}$, which is choice C.

Example 11

$$\sin(x) = \cos(K - x)$$

In the equation above, the angle measures are in radians and *K* is a constant. Which of the following could be the value of *K*?

A) 0

B) $\frac{\pi}{4}$

C) $\frac{\pi}{2}$

D) π

The complementary angle relationship for sine and cosine implies that the equation $\sin(x) = \cos(K - x)$ holds if *K* = 90°. Since 90° = $\frac{2\pi}{360°}$ × 90° = $\frac{\pi}{2}$ radians, the value of *K* could be $\frac{\pi}{2}$, which is choice C.

Complex Numbers

The SAT Math Test includes questions on the arithmetic of complex numbers.

The square of any real number is nonnegative. The number *i* is defined to be the solution to the equation $x^2 = -1$. That is, $i^2 = -1$, or $i = \sqrt{-1}$. Note that $i^3 = i^2(i) = -i$ and $i^4 = i^2(i^2) = -1(-1) = 1$.

A complex number is a number of the form $a + bi$, where *a* and *b* are real number constants and $i = \sqrt{-1}$. This is called the standard form of a complex number. The number *a* is called the real part of $a + bi$, and the number *b* is called the imaginary part of $a + bi$.

Addition and subtraction of complex numbers are performed by adding their real and complex parts. For example,

- $(-3 - 2i) + (4 - i) = (-3 + 4) + (-2i + (-i)) = 1 - 3i$
- $(-3 - 2i) - (4 - i) = (-3 - 4) + (-2i - (-i)) = -7 - i$

Multiplication of complex numbers is performed similarly to multiplication of binomials, using the fact that $i^2 = -1$. For example,

$$(-3 - 2i)(4 - i) = (-3)(4) + (-3)(-i) + (-2i)(4) + (-2i)(-i)$$

$$= -12 + 3i - 8i + (-2)(-1)i^2$$

$$= -12 - 5i + 2i^2$$

$$= -12 - 5i + 2(-1)$$

$$= -14 - 5i$$

☞ **REMEMBER**

The number *i* is defined to be the solution to equation $x^2 = -1$. Thus, $i^2 = -1$, and $i = \sqrt{-1}$.

☞ **REMEMBER**

If you have little experience working with complex numbers, practice adding, subtracting, multiplying, and dividing complex numbers until you are comfortable doing so. You may see complex numbers on the SAT Math Test.

The complex number $a - bi$ is called the conjugate of $a + bi$. The product of $a + bi$ and $a - bi$ is $a^2 - abi + abi - b^2i^2$; this reduces to $a^2 + b^2$, a real number. The fact that the product of a complex number and its conjugate is a real number can be used to perform division of complex numbers.

$$\frac{-3 - 2i}{4 - i} = \frac{-3 - 2i}{4 - i} \times \frac{4 + i}{4 + i}$$

$$= \frac{(-3 - 2i)(4 + i)}{(4 - i)(4 + i)}$$

$$= \frac{-12 - 3i - 8i - 2i^2}{4^2 - i^2}$$

$$= \frac{-10 - 11i}{17}$$

$$= -\frac{10}{17} - \frac{11}{17}i$$

Example 12

In the complex number system, which of the following is equal to $\frac{1 + i}{1 - i}$?
(Note: $i = \sqrt{-1}$)

A) i

B) $2i$

C) $-1 + i$

D) $1 + i$

Multiply both the numerator and denominator of $\frac{1 + i}{1 - i}$ by $1 + i$ to remove i from the denominator.

$$\frac{1 + i}{1 - i} = \frac{1 + i}{1 - i} \times \frac{1 + i}{1 + i}$$

$$= \frac{(1 + i)(1 + i)}{(1 - i)(1 + i)}$$

$$= \frac{1 + 2i + i^2}{1^2 - i^2}$$

$$= \frac{1 + 2i - 1}{1 - (-1)}$$

$$= \frac{2i}{2}$$

$$= i$$

Choice A is the correct answer.

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