Passport to Advanced Math

Passport to Advanced Math questions include topics that are especially important for students to master *before* studying advanced math. Chief among these topics is the understanding of the structure of expressions and the ability to analyze, manipulate, and rewrite these expressions. These questions also include reasoning with more complex equations and interpreting and building functions.

Heart of Algebra questions focus on the mastery of linear equations, systems of linear equations, and linear functions. In contrast, Passport to Advanced Math questions focus on the ability to work with and analyze more complex equations. The questions may require you to demonstrate procedural skill in adding, subtracting, and multiplying polynomials and in factoring polynomials. You may be required to work with expressions involving exponentials, integer and rational exponents, radicals, or fractions with a variable in the denominator. The questions may ask you to solve quadratic, radical, rational, polynomial, or absolute value equations. They may also ask you to solve a system consisting of a linear equation and a nonlinear equation. You may be required to manipulate an equation in several variables to isolate a quantity of interest.

Some questions in Passport to Advanced Math will ask you to build a quadratic or exponential function or an equation that describes a context or to interpret the function, the graph of the function, or the solution to the equation in terms of the context.

Passport to Advanced Math questions may assess your ability to recognize structure. Expressions and equations that appear complex may use repeated terms or repeated expressions. By noticing these patterns, the complexity of a problem can be reduced. Structure may be used to factor or otherwise rewrite an expression, to solve a quadratic or other equation, or to draw conclusions about the context represented by an expression, equation, or function. You may be asked to identify or derive the form of an expression, equation, or function that reveals information about the expression, equation, or function or the context it represents.

REMEMBER

16 of the 58 questions (28%) on the SAT Math Test are Passport to Advanced Math questions.

Passport to Advanced Math questions also assess your understanding of functions and their graphs. A question may require you to demonstrate your understanding of function notation, including interpreting an expression where the argument of a function is an expression rather than a variable. The questions may assess your understanding of how the algebraic properties of a function relate to the geometric characteristics of its graph.

Passport to Advanced Math questions include both multiple-choice questions and student-produced response questions. Some of these questions are in the no-calculator portion, where the use of a calculator is not permitted, and others are in the calculator portion, where the use of a calculator is permitted. When you can use a calculator, you must decide whether using your calculator is an effective strategy for that particular question.

Passport to Advanced Math is one of the three SAT Math Test subscores, reported on a scale of 1 to 15.

Let's consider the content and skills assessed by Passport to Advanced Math questions.

Operations with Polynomials and Rewriting Expressions

Questions on the SAT Math Test may assess your ability to add, subtract, and multiply polynomials.

Example 1

$$(x^2 + bx - 2)(x + 3) = x^3 + 6x^2 + 7x - 6$$

In the equation above, b is a constant. If the equation is true for all values of x, what is the value of b?

A) 2

B) 3

C) 7

D) 9

To find the value of b, use the distributive property to expand the left-hand side of the equation and then collect like terms so that the left-hand side is in the same form as the right-hand side.

$$(x^2 + bx - 2)(x + 3) = (x^3 + bx^2 - 2x) + (3x^2 + 3bx - 6)$$
$$= x^3 + (3 + b)x^2 + (3b - 2)x - 6$$

👉
REMEMBER

Passport to Advanced Math questions build on the knowledge and skills tested on Heart of Algebra questions. Develop proficiency with Heart of Algebra questions before tackling Passport to Advanced Math questions.

Since the two polynomials are equal for all values of x, the coefficient of matching powers of x should be the same. Therefore, comparing the coefficients of $x^3 + (3 + b)x^2 + (3b - 2)x - 6$ and $x^3 + 6x^2 + 7x - 6$ reveals that $3 + b = 6$ and $3b - 2 = 7$. Solving either of these equations gives $b = 3$, which is choice B.

Questions may also ask you to use structure to rewrite expressions. The expression may be of a particular type, such as a difference of squares, or it may require insightful analysis.

Example 2

> Which of the following is equivalent to $16s^4 - 4t^2$?
>
> A) $4(s^2 - t)(4s^2 + t)$
>
> B) $4(4s^2 - t)(s^2 + t)$
>
> C) $4(2s^2 - t)(2s^2 + t)$
>
> D) $(8s^2 - 2t)(8s^2 + 2t)$

A closer look reveals that the given equation follows the difference of two perfect squares pattern, $x^2 - y^2$, which factors as $(x - y)(x + y)$. The expression $16s^4 - 4t^2$ is also the difference of two squares: $16s^4 - 4t^2 = (4s^2)^2 - (2t)^2$. Therefore, it can be factored as $(4s^2)^2 - (2t)^2 = (4s^2 - 2t)(4s^2 + 2t)$. This expression can be rewritten as $(4s^2 - 2t)(4s^2 + 2t) = 2(2s^2 - t)(2)(2s^2 + t) = 4(2s^2 - t)(2s^2 + t)$, which is choice C.

Alternatively, a 4 could be factored out of the given equation: $4(4s^4 - t^2)$. The expression inside the parentheses is a difference of two squares. Therefore, it can be further factored as $4(2s^2 + t)(2s^2 - t)$.

Example 3

> Which expression is equivalent to $xy^2 + 2xy^2 + 3xy$?
>
> A) $2xy^2 + 3xy$
>
> B) $3xy^2 + 3xy$
>
> C) $6xy^4$
>
> D) $6xy^5$

There are three terms in the expression, the first two of which are like terms. The like terms can be added together by adding their coefficients: $xy^2 + 2xy^2 + 3xy = (xy^2 + 2xy^2) + 3xy$, which is equivalent to $3xy^2 + 3xy$. Therefore choice B is correct.

Quadratic Functions and Equations

Questions in Passport to Advanced Math may require you to build a quadratic function or an equation to represent a context.

Example 4

A car is traveling at x feet per second. The driver sees a red light ahead, and after 1.5 seconds reaction time, the driver applies the brake. After the brake is applied, the car takes $\frac{x}{24}$ seconds to stop, during which time the average speed of the car is $\frac{x}{2}$ feet per second. If the car travels 165 feet from the time the driver saw the red light to the time it comes to a complete stop, which of the following equations can be used to find the value of x?

A) $x^2 + 48x - 3{,}960 = 0$

B) $x^2 + 48x - 7{,}920 = 0$

C) $x^2 + 72x - 3{,}960 = 0$

D) $x^2 + 72x - 7{,}920 = 0$

During the 1.5-second reaction time, the car is still traveling at x feet per second, so it travels a total of $1.5x$ feet. The average speed of the car during the $\frac{x}{24}$-second braking interval is $\frac{x}{2}$ feet per second, so over this interval, the car travels $\left(\frac{x}{2}\right)\left(\frac{x}{24}\right) = \frac{x^2}{48}$ feet. Since the total distance the car travels from the time the driver saw the red light to the time it comes to a complete stop is 165 feet, you have the equation $\frac{x^2}{48} + 1.5x = 165$. This quadratic equation can be rewritten in standard form by subtracting 165 from each side and then multiplying each side by 48, giving $x^2 + 72x - 7{,}920 = 0$, which is choice D.

Some questions on the SAT Math Test will ask you to solve a quadratic equation. You must determine the appropriate procedure: factoring, completing the square, using the quadratic formula, using a calculator (if permitted), or using structure. You should also know the following facts in addition to the formulas in the directions:

- The sum of the solutions of $x^2 + bx + c = 0$ is $-b$.

- The product of the solutions of $x^2 + bx + c = 0$ is c.

Each of the facts can be seen from the factored form of a quadratic. If r and s are the solutions of $x^2 + bx + c = 0$, then $x^2 + bx + c = (x - r)(x - s)$. Thus, $b = -(r + s)$ and $c = (-r)(-s) = rs$. **Note:** To use either of these facts, the coefficient of x^2 must be equal to 1.

REMEMBER

The SAT Math Test may ask you to solve a quadratic equation. Be prepared to use the appropriate method. Practice using the various methods (below) until you are comfortable with all of them.

1. Factoring
2. Completing the square
3. Using the quadratic formula
4. Using a calculator
5. Using structure

Example 5

What are the solutions to the equation $x^2 - 3 = x$?

A) $\dfrac{-1 \pm \sqrt{11}}{2}$

B) $\dfrac{-1 \pm \sqrt{13}}{2}$

C) $\dfrac{1 \pm \sqrt{11}}{2}$

D) $\dfrac{1 \pm \sqrt{13}}{2}$

The equation can be solved by using the quadratic formula or by completing the square. Let's use the quadratic formula. First, subtract x from each side of $x^2 - 3 = x$ to put the equation in standard form: $x^2 - x - 3 = 0$. The quadratic formula states the solutions x of the equation $ax^2 + bx + c = 0$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. For the equation $x^2 - x - 3 = 0$, you have $a = 1$, $b = -1$, and $c = -3$. Substituting these values into the quadratic formula gives $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} = \dfrac{1 \pm \sqrt{1 - (-12)}}{2} = \dfrac{1 \pm \sqrt{13}}{2}$, which is choice D.

Example 6

If $x > 0$ and $2x^2 + 3x - 2 = 0$, what is the value of x?

The left-hand side of the equation can be factored: $2x^2 + 3x - 2 = (2x - 1)(x + 2) = 0$. Therefore, either $2x - 1 = 0$, which gives $x = \dfrac{1}{2}$, or $x + 2 = 0$, which gives $x = -2$. Since $x > 0$, the value of x is $\dfrac{1}{2}$.

REMEMBER

Pay close attention to all of the details in the question. In Example 6, x can equal $\dfrac{1}{2}$ or -2, but since the question states that $x > 0$, the value of x must be $\dfrac{1}{2}$.

Example 7

What is the sum of the solutions of $(2x - 1)^2 = (x + 2)^2$?

If a and b are real numbers and $a^2 = b^2$, then either $a = b$ or $a = -b$. Since $(2x - 1)^2 = (x + 2)^2$, either $2x - 1 = x + 2$ or $2x - 1 = -(x + 2)$. In the first case, $x = 3$, and in the second case, $3x = -1$, or $x = -\dfrac{1}{3}$. Therefore, the sum of the solutions x of $(2x - 1)^2 = (x + 2)^2$ is $3 + \left(-\dfrac{1}{3}\right) = \dfrac{8}{3}$.

Exponential Functions, Equations, and Expressions and Radicals

We examined exponential functions in Example 7 of Chapter 17. Some Passport to Advanced Math questions ask you to build a function that models a given context. As discussed in Chapter 17, exponential functions model situations in which a quantity is multiplied by a constant factor for each time period. An exponential function can be increasing with time, in which case it models exponential growth, or it can be decreasing with time, in which case it models exponential decay.

Example 8

A researcher estimates that the population of a city is increasing at an annual rate of 0.6%. If the current population of the city is 80,000, which of the following expressions appropriately models the population of the city t years from now according to the researcher's estimate?

A) $80,000(1 + 0.006)^t$

B) $80,000(1 + 0.006^t)$

C) $80,000 + 1.006^t$

D) $80,000(0.006^t)$

According to the researcher's estimate, the population is increasing by 0.6% each year. Since 0.6% is equal to 0.006, after the first year, the population is $80,000 + 0.006(80,000) = 80,000(1 + 0.006)$. After the second year, the population is $80,000(1 + 0.006) + 0.006(80,000)(1 + 0.006) = 80,000(1 + 0.006)^2$. Similarly, after t years, the population will be $80,000(1 + 0.006)^t$ according to the researcher's estimate. This is choice A.

A well-known example of exponential decay is the decay of a radioactive isotope. One example is iodine-131, a radioactive isotope used in some medical treatments, which decays to xenon-131. The half-life of iodine-131 is 8.02 days; that is, after 8.02 days, half of the iodine-131 in a sample will have decayed to xenon-131. Suppose a sample with a mass of A milligrams of iodine-131 decays for d days. Every 8.02 days, the quantity of iodine-131 is multiplied by $\frac{1}{2}$, or 2^{-1}. In d days, a total of $\frac{d}{8.02}$ different 8.02-day periods will have passed, and so the original quantity will have been multiplied by 2^{-1} a total of $\frac{d}{8.02}$ times. Therefore, the mass, in milligrams, of iodine-131 remaining in the sample will be $A(2^{-1})^{\frac{d}{8.02}} = A\left(2^{-\frac{d}{8.02}}\right)$.

In the preceding discussion, we used the identity $\frac{1}{2} = 2^{-1}$. Questions on the SAT Math Test may require you to apply this and other laws of exponents and the relationship between powers and radicals.

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A quantity that grows or decays by a fixed percent at regular intervals is said to possess exponential growth or decay, respectively.

Exponential growth is represented by the function $y = a(1 + r)^t$, while exponential decay is represented by the function $y = a(1 - r)^t$, where y is the new population, a is the initial population, r is the rate of growth or decay, and t is the number of time intervals that have elapsed.

Some Passport to Advanced Math questions ask you to use properties of exponents to rewrite expressions.

Example 9

Which of the following is equivalent to $\left(\dfrac{1}{\sqrt{x}}\right)^{n}$?

A) $x^{\frac{n}{2}}$

B) $x^{-\frac{n}{2}}$

C) $x^{n+\frac{1}{2}}$

D) $x^{n-\frac{1}{2}}$

The expression $\sqrt{x}$ is equal to $x^{\frac{1}{2}}$. Thus, $\dfrac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$,

and $\left(\dfrac{1}{\sqrt{x}}\right)^{n} = \left(x^{-\frac{1}{2}}\right)^{n} = x^{-\frac{n}{2}}$. Choice B is the correct answer.

An SAT Math Test question may also ask you to solve a radical equation. In solving radical equations, you may square both sides of an equation. Since squaring both sides of an equation may result in an extraneous solution, you may end up with a root to the simplified equation that is not a root to the original equation. Thus, when solving a radical equation, you should check any solution you get in the original equation.

Example 10

$$x - 12 = \sqrt{x + 44}$$

What are all possible solutions to the given equation?

A) 5

B) 20

C) −5 and 20

D) 5 and 20

Squaring each side of $x - 12 = \sqrt{x + 44}$ gives

$$(x - 12)^2 = (\sqrt{x + 44})^2 = x + 44$$

$$x^2 - 24x + 144 = x + 44$$

$$x^2 - 25x + 100 = 0$$

$$(x - 5)(x - 20) = 0$$

The solutions to the quadratic equation are $x = 5$ and $x = 20$. However, since the first step was to square each side of the given equation, which may have introduced an extraneous solution, you need to check $x = 5$ and $x = 20$ in the original equation. Substituting 5 for x gives

$$5 - 12 = \sqrt{5 + 44}$$

$$-7 = \sqrt{49}$$

This is not a true statement (since $\sqrt{49}$ represents the principal square root, or only the positive square root, 7), so $x = 5$ is *not* a solution to $x - 12 = \sqrt{x + 44}$. Substituting 20 for x gives

$$20 - 12 = \sqrt{20 + 44}$$

$$8 = \sqrt{64}$$

This is a true statement, so $x = 20$ is a solution to $x - 12 = \sqrt{x + 44}$. Therefore, the only solution to the given equation is 20, which is choice B.

Solving Rational Equations

Questions on the SAT Math Test may assess your ability to work with rational expressions, including fractions with a variable in the denominator. This may include finding the solution to a rational equation.

Example 11

$$\frac{3}{t + 1} = \frac{2}{t + 3} + \frac{1}{4}$$

If t is a solution to the given equation and $t > 0$, what is the value of t?

If both sides of the equation are multiplied by the lowest common denominator, which is $4(t + 1)(t + 3)$, the resulting equation will not have any fractions, and the variable will no longer be in the denominator. This gives $12(t + 3) = 8(t + 1) + (t + 1)(t + 3)$. This can be rewritten as $12t + 36 = (8t + 8) + (t^2 + 4t + 3)$, or $12t + 36 = t^2 + 12t + 11$, which simplifies to $0 = t^2 - 25$. This equation factors to $0 = (t - 5)(t + 5)$. Therefore, the solutions to the equation are $t = 5$ and $t = -5$. Since $t > 0$, the value of t is 5.

Systems of Equations

Questions on the SAT Math Test may ask you to solve a system of equations in two variables in which one equation is linear and the other equation is quadratic or another nonlinear equation.

Example 12

$$3x + y = -3$$

$$(x + 1)^2 - 4(x + 1) - 6 = y$$

If (x, y) is a solution of the system of equations above and $y > 0$, what is the value of y?

One method for solving systems of equations is substitution. If the first equation is solved for y, it can be substituted in the second equation. Subtracting $3x$ from each side of the first equation gives you $y = -3 - 3x$, which can be rewritten as $y = -3(x + 1)$.
Substituting $-3(x + 1)$ for y in the second equation gives you

$(x + 1)^2 − 4(x + 1) − 6 = −3(x + 1)$. Since the factor $(x + 1)$ appears as a squared term and a linear term, the equation can be thought of as a quadratic equation in the variable $(x + 1)$, so collecting the terms and setting the expression equal to 0 gives you $(x + 1)^2 − (x + 1) − 6 = 0$. Factoring gives you $((x + 1) − 3)((x + 1) + 2) = 0$, or $(x − 2)(x + 3) = 0$. Thus, either $x = 2$, which gives $y = −3 − 3(2) = −9$; or $x = −3$, which gives $y = −3 − 3(−3) = 6$. Therefore, the solutions to the system are $(2, −9)$ and $(−3, 6)$. Since the question states that $y > 0$, the value of y is 6.
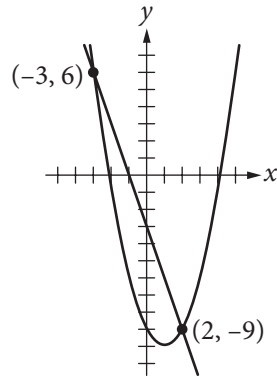
The solutions of the system are given by the intersection points of the two graphs. Questions on the SAT Math Test may assess this or other relationships between algebraic and graphical representations of functions.

Relationships Between Algebraic and Graphical Representations of Functions

A function f has a graph in the xy-plane, which is the graph of the equation $y = f(x)$, or, equivalently, consists of all ordered pairs $(x, f(x))$. Some Passport to Advanced Math questions assess your ability to relate properties of the function f to properties of its graph, and vice versa. You may be required to apply some of the following relationships:

- **Intercepts.** The x-intercepts of the graph of f correspond to values of x such that $f(x) = 0$, which corresponds to where the graph intersects with the x-axis; if the function f has no zeros, its graph has no x-intercepts, and vice versa. The y-intercept of the graph of f corresponds to the value of $f(0)$, or where the graph intersects with the y-axis. If $x = 0$ is not in the domain of f, the graph of f has no y-intercept, and vice versa.

- **Domain and range**. The domain of f is the set of all x for which $f(x)$ is defined. The range of f is the set of all y such that $y = f(x)$ for some value of x in the domain. The domain and range can be found from the graph of f as the set of all x-coordinates and y-coordinates, respectively, of points on the graph.

- **Maximum and minimum values**. The maximum and minimum values of f can be found by locating the highest and the lowest points on the graph, respectively. For example, suppose P is the

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The domain of a function is the set of all values for which the function is defined. The range of a function is the set of all values that correspond to the values in the domain, given the relationship defined by the function, or the set of all outputs that are associated with all of the possible inputs.

highest point on the graph of *f*. Then the *y*-coordinate of *P* is the maximum value of *f*, and the *x*-coordinate of *P* is where *f* takes on its maximum value.

▪ **Increasing and decreasing.** The graph of *f* shows the intervals over which the function *f* is increasing and decreasing.

▪ **End behavior.** The graph of *f* can indicate if *f*(*x*) increases or decreases without limit as *x* increases or decreases without limit.

▪ **Transformations.** For a graph of a function *f*, a change of the form *f*(*x*) + *a* will result in a vertical shift of *a* units and a change of the form *f*(*x* + *a*) will result in a horizontal shift of *a* units.

Note: The SAT Math Test uses the following conventions about graphs in the *xy*-plane *unless* a particular question clearly states or shows a different convention:

▪ The axes are perpendicular.

▪ Scales on the axes are linear scales.

▪ The size of the units on the two axes *cannot* be assumed to be equal unless the question states they are equal or you are given enough information to conclude they are equal.

▪ The values on the horizontal axis increase as you move to the right.

▪ The values on the vertical axis increase as you move up.

REMEMBER

Don't assume the size of the units on the two axes are equal unless the question states they are equal or you can conclude they are equal from the information given.

Example 13

The graph of which of the following functions in the *xy*-plane has *x*-intercepts at −4 and 5?

A) $f(x) = (x + 4)(x - 5)$

B) $g(x) = (x - 4)(x + 5)$

C) $h(x) = (x - 4)^2 + 5$

D) $k(x) = (x + 5)^2 - 4$

The *x*-intercepts of the graph of a function correspond to the zeros of the function. All the functions in the choices are defined by quadratic equations, so the answer must be a quadratic function. If a quadratic function has *x*-intercepts at −4 and 5, then the values of the function at −4 and 5 are each 0; that is, the zeros of the function occur at $x = -4$ and at $x = 5$. Since the function is defined by a quadratic equation and has zeros at $x = -4$ and $x = 5$, it must have $(x + 4)$ and $(x - 5)$ as factors. Therefore, choice A, $f(x) = (x + 4)(x - 5)$, is correct.

The graph in the xy-plane of each of the functions in the previous example is a parabola. Using the defining equations, you can tell that the graph of g has x-intercepts at 4 and −5; the graph of h has its vertex at (4, 5); and the graph of k has its vertex at (−5, −4).

Example 14
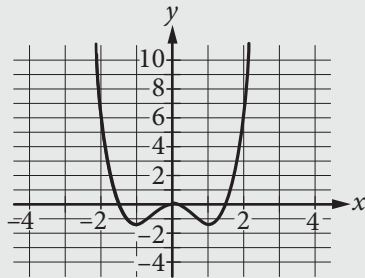
The function $f(x) = x^4 - 2.4x^2$ is graphed in the xy-plane where $y = f(x)$, as shown above. If k is a constant such that the equation $f(x) = k$ has 4 solutions, which of the following could be the value of k?

A) 1

B) 0

C) −1

D) −2

Choice C is correct. Since $f(x) = x^4 - 2.4x^2$, the equation $f(x) = k$, or $x^4 - 2.4x^2 = k$, will have 4 solutions if and only if the graph of the horizontal line with equation $y = k$ intersects the graph of f at 4 points. The graph shows that of the given choices, only for choice C, −1, does the graph of $y = −1$ intersect the graph of f at 4 points.

Function Notation

The SAT Math Test assesses your understanding of function notation. You must be able to evaluate a function given the rule that defines it, and if the function describes a context, you may need to interpret the value of the function in the context. A question may ask you to interpret a function when an expression, such as $2x$ or $x + 1$, is used as the argument instead of the variable x.

Example 15

If $g(x) = 2x + 1$ and $f(x) = g(x) + 4$, what is $f(2)$?

You are given $f(x) = g(x) + 4$ and therefore $f(2) = g(2) + 4$. To determine the value of $g(2)$, use the function $g(x) = 2x + 1$. Thus, $g(2) = 2(2) + 1$, and therefore $g(2) = 5$. Substituting $g(2)$ gives $f(2) = 5 + 4$, or $f(2) = 9$.

Alternatively, since $f(x) = g(x) + 4$ and $g(x) = 2x + 1$, it follows that $f(x)$ must equal $2x + 1 + 4$, or $2x + 5$. Therefore, $f(2) = 2(2) + 5 = 9$.

Interpreting and Analyzing More Complex Equations in Context

Equations and functions that describe a real-life context can be complex. Often, it's not possible to analyze them as completely as you can analyze a linear equation or function. You still can acquire key information about the context by interpreting and analyzing the equation or function that describes it. Passport to Advanced Math questions may ask you to identify connections between the function, its graph, and the context it describes. You may be asked to use an equation describing a context to determine how a change in one quantity affects another quantity. You may also be asked to manipulate an equation to isolate a quantity of interest on one side of the equation. You may be asked to produce or identify a form of an equation that reveals new information about the context it represents or about the graphical representation of the equation.

Example 16

> For a certain reservoir, the function f gives the water level $f(n)$, to the nearest whole percent of capacity, on the nth day of 2016. Which of the following is the best interpretation of $f(37) = 70$?
>
> A) The water level of the reservoir was at 37% capacity for 70 days in 2016.
>
> B) The water level of the reservoir was at 70% capacity for 37 days in 2016.
>
> C) On the 37th day of 2016, the water level of the reservoir was at 70% capacity.
>
> D) On the 70th day of 2016, the water level of the reservoir was at 37% capacity.

The function f gives the water level, to the nearest whole percent of capacity, on the nth day of 2016. It follows that $f(37) = 70$ means that on the 37th day of 2016, the water level of the reservoir was at 70% capacity. This statement is choice C.

Example 17

> If an object of mass m is moving at speed v, the object's kinetic energy (KE) is given by the equation $KE = \frac{1}{2} mv^2$. If the mass of the object is halved and its speed is doubled, how does the kinetic energy change?
>
> A) The kinetic energy is halved.
>
> B) The kinetic energy is unchanged.
>
> C) The kinetic energy is doubled.
>
> D) The kinetic energy is quadrupled (multiplied by a factor of 4).

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Another way to check your answer in Example 17 is to substitute simple numerical values for the variables m, v, and KE and examine the effect on KE when those values are altered as indicated by the question. If the value 1 is substituted for both m and v, then KE is $\frac{1}{2}$. However, substituting $\frac{1}{2}$ for m and 2 for v yields a value for KE of 1. Since 1 is twice the value of $\frac{1}{2}$, you know that KE is doubled.

Choice C is correct. If the mass of the object is halved, the new mass is $\frac{m}{2}$. If the speed of the object is doubled, its new speed is $2v$. Therefore, the new kinetic energy is $\frac{1}{2}\left(\frac{m}{2}\right)(2v)^2 = \frac{1}{2}\left(\frac{m}{2}\right)(4v^2) = mv^2$. This is double the original kinetic energy of the object, which was $\frac{1}{2}mv^2$.

Example 18

A gas in a container will escape through holes of microscopic size, as long as the holes are larger than the gas molecules. This process is called effusion. If a gas of molar mass M_1 effuses at a rate of r_1 and a gas of molar mass M_2 effuses at a rate of r_2, then the following relationship holds.

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

This is known as Graham's law. Which of the following correctly expresses M_2 in terms of M_1, r_1, and r_2?

A) $M_2 = M_1\left(\dfrac{r_1^2}{r_2^2}\right)$

B) $M_2 = M_1\left(\dfrac{r_2^2}{r_1^2}\right)$

C) $M_2 = \sqrt{M_1}\left(\dfrac{r_1}{r_2}\right)$

D) $M_2 = \sqrt{M_1}\left(\dfrac{r_2}{r_1}\right)$

Squaring each side of $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$ gives $\left(\frac{r_1}{r_2}\right)^2 = \left(\sqrt{\frac{M_2}{M_1}}\right)^2$, which can be rewritten as $\frac{M_2}{M_1} = \frac{r_1^2}{r_2^2}$. Multiplying each side of $\frac{M_2}{M_1} = \frac{r_1^2}{r_2^2}$ by M_1 gives $M_2 = M_1\left(\frac{r_1^2}{r_2^2}\right)$, which is choice A.

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Always start by identifying exactly what the question asks. In Example 18, you are being asked to isolate the variable M_2. Squaring both sides of the equation is a great first step as it allows you to eliminate the radical sign.

Example 19

A store manager estimates that if a video game is sold at a price of p dollars, the store will have weekly revenue, in dollars, of $r(p) = -4p^2 + 200p$ from the sale of the video game. Which of the following equivalent forms of $r(p)$ shows, as constants or coefficients, the maximum possible weekly revenue and the price that results in the maximum revenue?

A) $r(p) = 200p - 4p^2$

B) $r(p) = -2(2p^2 - 100p)$

C) $r(p) = -4(p^2 - 50p)$

D) $r(p) = -4(p - 25)^2 + 2{,}500$

Choice D is correct. The graph of *r* in the coordinate plane is a parabola that opens downward. The maximum value of revenue corresponds to the vertex of the parabola. Since the square of any real number is always nonnegative, the form $r(p) = -4(p - 25)^2 + 2{,}500$ shows that the vertex of the parabola is (25, 2,500); that is, the maximum must occur where $-4(p - 25)^2$ is 0, which is $p = 25$, and this maximum is $r(25) = 2{,}500$. Thus, the maximum possible weekly revenue and the price that results in the maximum revenue occur as constants in the form $r(p) = -4(p - 25)^2 + 2{,}500$.